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Bispectral Analysis for non-Gaussian winds: how and when?

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SUMMARY:
The non Gaussian nature of turbulent wind loading has been long accepted. Nevertheless, although this might have significant influence on design quantities, it seems to be very often ignored when it comes to practical applications, specially when dealing with real structures, ranging from medium to quite large dimensions. Unfortunately, the need for significantly high computational power has been the main challenge to overcome. However, things have evolved quite fast in that domain over the last few years. This work aims at answering a quite as important as probably still unanswered question: although wind loading in most cases happens to be non Gaussian, when exactly can't non Gaussianity be neglected ?

Keywords: buffeting, non-Gaussian, stochastics

1. INTRODUCTION

Natural actions always existed, and always will. Natural hazard "naturally" comes with them. And today, we are experiencing it much more frequently than a decade ago. Floods, tornadoes, rainstorms, extreme winds, extreme temperatures, just to mention some. Some might call for irreversible climate change. Others would argue that the most part of this is just the natural cycle of the Earth system, as history teaches us. Of course, science is working hard to give an accurate answer. What it is sure, without any doubt, is that engineers must take these increasingly happening extreme phenomena into account, for new as well as existing structures. Wind is certainly part of this. Many studies are being carried out on downbursts phenomena, cyclonic winds, tornadoes, and extreme winds in order to try to better understand them and consequently provide some models to better account for them (Solari, 2020). These are highly non-linear, non-stationary, non-gaussian (also referred as the three Non's (Kareem and Wu, 2013)) phenomena, which current state of the art, specially for practical applications, is not yet fully adapted to. Since it might seem, reasonably, too hard at first to consider all these non-common physical behaviours at once, considering them one by one might be the key to find a general approach on how to deal with them. In this context, our focus is on stationary, non-Gaussian wind loads, and the effect that these, compared to their Gaussian assumption counterpart, have on the determination of the stochastic dynamic response.

2. THEORETICAL BACKGROUND

In the most general case, buffeting analysis is governed by the Multi-Degree-Of-Freedom equations of motion, which reads:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{1}$$

where, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are structural mass, damping, stiffness matrix respectively, $\mathbf{x}(t)$ denotes the dynamic system's response, $\mathbf{f}(t)$ the relative applied wind load.

There are two fundamental ways to solve this problem: (i) wind sample generation and deterministic Monte Carlo time-domain analysis, if measurements/simulations (Gioffrè et al., 2000) of the non-Gaussian wind load $\mathbf{f}(t)$ are available; (ii) stochastic frequency-domain analysis, if probabilistic description of the loading is known, as it is quite always the case for buffeting analysis. Indeed, each method has its own advantages and disadvantages. Time domain resolution well fits when system exhibits highly non-linear behaviour. On the other hand, stochastic spectral analysis (Denoël, 2015; Denoël et al., 2023) might provide a faster solution, which also better highlights the physical load-structure interaction. First applications in Wind Engineering date back to DAvenport's works in which the Power Spectral Density function (PSD) of the response is expressed as the product of the system's transfer function and the PSD of the random applied load,

$$\mathbf{S}_{\mathbf{x}}(\omega) = \mathbf{H}(\omega)\, \mathbf{S}_{\mathbf{f}}(\omega)\, \mathbf{H}^{*}(\omega) \tag{2}$$

where $\mathbf{H} = \left(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right)^{-1}$ is the Frequency Response Function (FRF) and $(\cdot)^*$ denotes the complex conjugate operator. Eq. (2) directly comes from the Frequency Domain equation of motion that reads $\mathbf{X}(\omega) = \mathbf{H}(\omega)\ \mathbf{P}(\omega)$, and from the property that $\mathbf{S}_{\mathbf{x}}(\omega) = \mathbf{X}(\omega)\ \mathbf{X}^{*}(\omega)$. An important property of the PSD function is that its integration in the 1D frequency space quantifies the second order statistical moment of the random process $\mathbf{x}(t)$:

$$\mathbf{m}_{2,\mathbf{x}} = \int_{-\infty}^{+\infty} \mathbf{S}_{\mathbf{x}}(\omega)\, \mathrm{d}\omega. \tag{3}$$

In probability, it is known that a Gaussian random process is fully characterised, in a probabilistic sense, if the first two statistical moments, namely mean and variance, are known. Therefore, a Spectral Analysis is sufficient in order to estimate systems' response, and eventually predict extreme values.

However, an infinite number of statistical descriptors would be necessary to accurately describe non-Gaussian processes. In practice, in Wind Engineering, an estimation of the 3$^{\mathrm{rd}}$ and 4$^{\mathrm{th}}$ order skewness and kurtosis coefficients is a good starting point.In this context, our focus is on the accurate, numerical evaluation of the (3$^{\mathrm{rd}}$ order) skewness coefficient $\gamma_{3,x} = m_{3,x}/m_{2,x}^{3/2}$ where $m_{3,x}$ is the third-order statistical moment, $m_{2,x}$ the second order as of Eq. (3), and

$$m_{3,\mathbf{x}} = \iint_{-\infty}^{+\infty} B_{\mathbf{x}}(\omega_1,\omega_2)\, \mathrm{d}\omega_1 \mathrm{d}\omega_2 \quad \text{and} \quad B_{\mathbf{x}}(\omega_1,\omega_2) = \mathbf{H}(\omega_1)\mathbf{H}(\omega_2)\mathbf{H}^{*}(\omega_1+\omega_2)B_{\mathbf{p}}(\omega_1,\omega_2) \tag{4}$$

where $B_{\mathbf{x}}(\omega_1,\omega_2)$ denotes the bispectrum of the non-Gaussian system's response, and $B_{\mathbf{f}}(\omega_1,\omega_2)$ the bispectrum of the non-Gaussian wind load.

3. WHY AND WHEN IS A NON-GAUSSIAN ANALYSIS REALLY NECESSARY

The only rigorous way to have a clear picture of whether the non-Gaussian nature of wind loading should not be neglected, all possible situations should be modeled and tested, such to build a database-like set of susceptible configurations. However, one of the main reasons why Bispectral analysis of structures under non-Gaussian winds has not yet fully taken place is its heavy computational cost, in terms of both time and memory storage, in particular for the evaluation of $m_{2,x}$ and $m_{3,x}$ (expressed as a single and a double-fold integral). In such context, a novel approach based on assuming distinct load-vs-structure timescales, which finds its roots in the Background-Resonant decomposition introduced by Davenport in the wind engineering community (Davenport, 1961), has been later developed for higher moments (Denoël, 2011, 2015), which helps reducing those integrals by at least one dimension. Furthermore, an optimised algorithm is being actively developed for a faster, accurate estimation of Eq. (4). Its main objective is providing a tool which makes Bispectral Analysis of structure a much more accessible and appealing approach, specially for real, medium-to-large-sized applications. Yet, once the algorithmic setup will be completed and fully optimised, it is considered the deployment of such methodology to real examples much more feasible, allowing for a much broader set of example to be carefully studied and categorised. Nevertheless, an illustrative example is reported, such that the readed can have a preview of the expected outcome of a Bispectral analysis of structures under non-Gaussian turbulent wind loading.

In Fig. 1, a studied model of a highway signaling panel as one shown in Fig. 2 is presented:

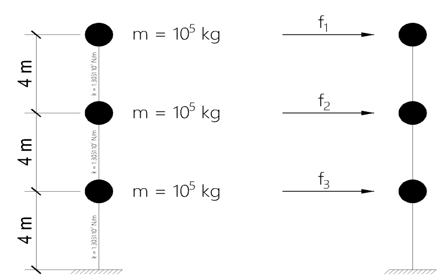

Figure 1. Discrete model example

Figure 2. Real highway signaling panel example.

4. CONCLUSIONS

The recent development of a bispectral analysis framework benefitting from an optimized algorithmic arrangement has made possible the parametric analysis of a small multi degree-of-freedom structure in order to determine the conditions under which a bispectral analysis is really necessary (since extreme/design values are significantly affected). It turns out that a third order statistical analysis is necessary when the non-Gaussianity (in terms of skewness coefficient) of the loading, which increases with wind turbulence in nonlinear buffeting models, is larger or similar to 0.3, for which one has, considering around 7% damping modal damping ratio, an average error of 5% between Gaussian vs. non-Gaussian in extremes values of the structural response. Of course, as already underlined, this is on purpose a very simple example, yet used with the main goal of encourage the reader understand that in real applications, these "small" numbers (which in such

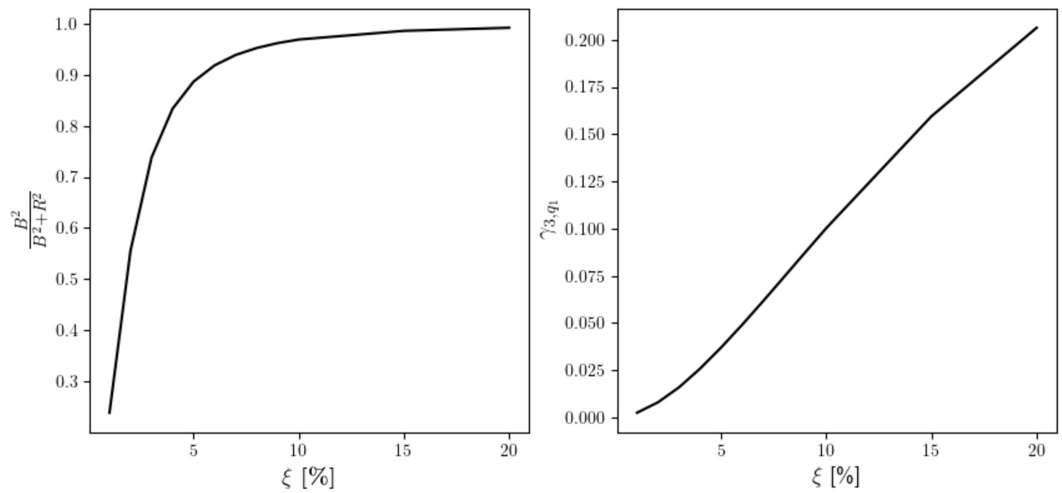

Figure 3. (a) Background to resonant ratio with increasing damping ratio. (b) Skewness of non-Gaussian modal response in mode 1, with increasing damping ratio.

case might not result in very dangerous outcomes), can make an important difference in terms of project design/verification. In future works, the same conclusions will be drawn for line-like structures (bridges and towers) as to the minimum spatial extent beyond which non-Gaussianity might be neglected.

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